Elementary Probability Theory

- Classical Defintion of Probabilty :

Suppose an event E can happen in h ways out of a total of n possible equally likely ways. Then the probability of occurance of the event (called its success) is denoted by :

$$P = Pr(E) = \frac{h}{n}$$

The probability of non-occurance of the event (called its failure) is denoted by :

$$q = Pr(\text{not } E) = \frac{n-h}{n} = 1 - \frac{h}{n}$$

$$= 1 - P = 1 - Pr(E)$$

Thus $p + q = 1$, OR

$$Pr(E) + Pr(\text{not } E) = 1$$

- Note: The event (not E) is sometimes denoted by $\bar{E}$.

Example: Let E be the event that the numbers 3 or 4 turn up in a single toss of a die. There are six ways in which the die can fall, resulting in the numbers 1, 2, 3, 4, 5 or 6; and if the die is fair, we can assume these six ways to be equallay likely. Since E can occure in two of these ways,

$\Rightarrow$ we have $p = Pr(E) = \frac{2}{6} = \frac{1}{3}$

The probability of not getting a 3 or 4 (i.e. getting a 1, 2, 5, or 6) is

$$q = Pr(\bar{E}) = 1 - \frac{1}{3} = \frac{2}{3}$$

Note that the probability of an event is a number between 0 and 1.

Example: Determine the probabilty P for each of the following events:

a) An odd number appears in a single toss of a fair die.

Solution: Out of 6 possible equally likely cases, 3 cases (where die comes up 1,3, or 5) are the event. Then:

$$P = \frac{3}{6} = \frac{1}{2}.$$

b) At least one head appears in two tosses of a fair coin.

Solution: If H denotes "head" an T denotes "tail", the two tosses can lead to 4 cases:

HH, HT, TH, TT all equally likely.

Only the first 3 cases are the event.

Then $P = \frac{3}{4}$.

c) The sum 7 appears in a single toss of a pair of a fair dice.

<u>solution</u>: Each of the 6 faces of one die can be associated with each of the 6 faces of the other die. so that the total number of cases which can arise, all equally likely, is:

$$(6)(6) = 36$$

These can be denoted by:

$(1,1)$, ~~(1,2)~~ $(2,1)$ $(3,1)$, ... $(6,6)$.

There are 6 ways of obtaining the sum 7, denoted by $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$

Then $p = \frac{6}{36} = \frac{1}{6}$

Example: A ball is drawn at random from a box containing 6 red balls, 4 white balls and 5 blue balls. Determine the probability that it is a) red
b) white
c) blue
d) not red
e) red or white.

Solution: Let R, W, B denotes the event of drawing, a red ball, white ball and blue ball respectively, then

a) $Pr(R) = \dfrac{\text{ways of chossing a red ball}}{\text{total ways of chossing a ball}}$.

$$= \frac{6}{6+4+5} = \frac{6}{15} = \frac{2}{5}$$

b) $Pr(W) = \dfrac{4}{6+4+5} = \dfrac{4}{15}$

c) $Pr(B) = \dfrac{6}{6+4+5} = \dfrac{5}{15} = \dfrac{1}{3}$

d) $Pr(\bar{R}) = 1 - Pr(R) = 1 - \frac{2}{5} = \frac{3}{5}$ by a)

e) $Pr(R+W) = \dfrac{\text{ways of chossing a red or white ball}}{\text{total ways of chossing a ball}}$

$$= \frac{6+4}{6+4+5} = \frac{10}{15} = \frac{2}{3}$$

Example: At a car park there are 100 vehicle, 60 of which are cars, 30 are vans and the remainder are lorries. if every vehicle is equally likely to leave. Find the probabilty of:

a) Van leaving first

b) lorry leaving first.

c) Car leaving second if either a lorry or van had left first.

Solution: a) Let S be the sample space and A be the event of a van leaving first.

$\therefore n(S) = 100$

$n(A) = 30$

$P(A) = \frac{30}{100} = \frac{3}{10}$

b) Let B be the event of a lorry leaving first.

$n(B) = 100 - 60 - 30 = 10$

$\therefore P(B) = \frac{10}{100} = \frac{1}{10}$

c) if either a lorry or a van had left first, then there would be 99 vehicles remaining, 60 of which are cars. Let T be the sample space and C be the event of a car leaving.

$n(T) = 99$

$n(C) = 60$

$\therefore$ Probability of a car leaving after a lorry or van has left is

$$P(c) = \frac{60}{99} = \frac{20}{33}$$

Example: A survay was taken on 30 classes at a school to find the total number of left-handed students in each class. The table below shows the results.

No. of left handed students X	0	1	2	3	4	5
frequency (no. of classes) f	1	2	5	12	8	2

A class was selected at random.

a) Find the probability that the class has 2 left handed students.

b) What is the probability that the class has at least 3 let-handed students?

c) Given that the total number of students in the 30 classes is 960, Find the Probability that a student randomly chosen from these 30 classes is left-handed?

Solution: a) Let S be the sample space and A be the event of a class having 2 left handed students.

$n(S) = 30$ and $n(A) = 5$

$\therefore P(A) = \frac{5}{30} = \frac{1}{6}$

b) Let B be the event of a class having at least 3 left handed students.

$n(B) = 12 + 8 + 2 = 22$

$P(B) = \frac{22}{30} = \frac{11}{15}$

c) First find the total number of left handed student

x	0	1	2	3	4	5
f	1	2	5	12	8	2
fx	0	2	10	36	32	10

Total no. of left handed students

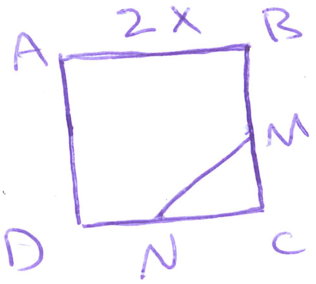
$= 2 + 10 + 36 + 32 + 10 = 90$

Here, the sample space is the total no. of students in the 30 clases, which was given as 960.

Let T be the sample space and c be the event that a student is left handed

$n(T) = 960$, $n(c) = 90$, $P(c) = \frac{90}{960} = \frac{3}{32}$

Example: ABCD is a square. M is the midpoint of BC and N is the midpoint of CD. A point is selected at random in the square. Calculate the probability that it lies in the triangle MCN



Solution:

Area of square $= 2x \times 2x$

$= 4x^2$

Area of triangle MCN is $\frac{1}{2}x^2$

$P(\text{point in the triangle}) = \frac{1}{2}x^2 \div 4x^2$

$= \frac{1}{2}x^2 \times \frac{1}{4x^2}$

$= \frac{1}{8}$